

I B. Tech II Semester Regular Examinations, April/May - 2017
MATHEMATICS-II (MM)
 (Com. to CE, EEE, ME, CHEM, AE, BIO, AME, MM, PE, PCE, MET)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **FOUR** Questions from **Part-B**
- ~~~~~

PART -A

1. a) Explain the Bisection method. (2M)
- b) Prove that $E = 1$. (2M)
- c) Write Newton's forward interpolation formula. (2M)
- d) Write Trapezoidal rule and Simpson's $3/8^{\text{th}}$ rule. (2M)
- e) Write the Fourier series for $f(x)$ in the interval $(0, 2\pi)$. (2M)
- f) Write One dimensional wave equation with boundary and initial conditions. (2M)
- g) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that (2M)

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

PART -B

2. a) Using bisection method, obtain an approximate root of the equation $x^3 - x - 1 = 0$. (7M)
- b) Develop an Iterative formula to find the square root of a positive number N using Newton-Raphson method. (7M)
3. a) Evaluate $\tan^{-1}(\tan^2 x)$. (6M)
- b) Using Newton's forward formula, find the value of $f(1.6)$, if (8M)

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5



4. a) Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x) dx$ using Simpson's $\frac{3}{8}$ rule. (7M)

b) Using the fourth order Runge – Kutta formula, find $y(0.2)$ and $y(0.4)$ given that (7M)

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$$

5. a) Find a Fourier series to represent $f(x) = x - x^2$ in $-\pi \leq x \leq \pi$. Hence show that (7M)

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}.$$

b) Obtain the half range sine series for $f(x) = e^x$ in $0 < x < 1$. (7M)

6. a) Solve by the method of separation of variables (7M)

$$4u_x + u_y = 3u \text{ and } u(0, y) = e^{-5y}.$$

b) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a (7M)

position given by $y = y_0 \sin^3 \frac{\pi x}{L}$ if it is released from rest from this position,

find the displacement $y(x, t)$.

7. a) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. Hence (7M)

$$\text{evaluate } \int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$$

b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ hence evaluate (7M)

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx.$$



I B. Tech II Semester Regular Examinations, April/May - 2017
MATHEMATICS-II (MM)
 (Com. to CE, EEE, ME, CHEM, AE, BIO, AME, MM, PE, PCE, MET)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **FOUR** Questions from **Part-B**

PART - A

1. a) Explain the Method of false position. (2M)
 - b) Prove that $\nabla = 1 - E^{-1}$. (2M)
 - c) Write Newton's backward interpolation formula. (2M)
 - d) Write Simpson's $1/3^{\text{rd}}$ and $3/8^{\text{th}}$ rule. (2M)
 - e) Write the Fourier series for $f(x)$ in the interval $(0, 2L)$. (2M)
 - f) Write the suitable solution of one dimensional wave equation. (2M)
 - g) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that (2M)
- $$F\{f(x-a)\} = e^{ias} F(s).$$

PART - B

2. a) Using bisection method, compute the real root of the equation $x^3 - 4x + 1 = 0$. (7M)
- b) Develop an Iterative formula to find the cube root of a positive number N using Newton-Raphson method. (7M)
3. a) Evaluate $\left(\frac{e^x}{\log 2x} \right)$. (6M)
- b) Using Newton's forward formula compute $f(142)$ from the following table: (8M)

x	140	150	160	170	180
$f(x)$	3.685	4.854	6.302	8.076	10.225

4. a) Evaluate, $\int_0^2 +e^{-x_2} dx$ by using Trapezoidal rule and Simpson's $\frac{1}{3}^{\text{rd}}$ rule taking (7M)
 - $h = 0.25$.
 - b) Find the value of y at $x = 0.1$ by Picard's method, given that (7M)
- $$\frac{dy}{dx} = \frac{y-x}{y+x}, \quad y(0) = 1.$$



5. a) Given that $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Find the Fourier series for $f(x)$. (7M)

Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$.

- b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (7M)
6. a) Solve by the method of separation of variables (7M)

$$\frac{\partial^2 u}{\partial x^2} = 2 \frac{\partial u}{\partial t} + u, \text{ given } u(x, 0) = 6e^{-3x}.$$

- b) A string of length L is initially at rest in equilibrium position and each of its points (7M)
is given the velocity $\frac{\partial y}{\partial t} \Big|_{t=0} = b \sin^3 \frac{\pi x}{L}$. Find displacement $y(x, t)$.

7. a) Express $f(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate (7M)

$$+ \int_0^{\infty} \frac{1 - \cos(\pi\lambda)}{\lambda} \sin(x\lambda) d\lambda.$$

- b) Find the Fourier sine and cosine transform (7M)
of $f(x) = e^{-ax}$, $a > 0$, $x > 0$.



I B. Tech II Semester Regular Examinations, April/May - 2017
MATHEMATICS-II (MM)
 (Com. to CE, EEE, ME, CHEM, AE, BIO, AME, MM, PE, PCE, MET)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **FOUR** Questions from **Part-B**

PART -A

1. a) Explain the Newton-Raphson method. (2M)
 - b) Prove that $\delta = E^{1/2} - E^{-1/2}$. (2M)
 - c) Write Lagrange's interpolation formula for unequal intervals. (2M)
 - Explain Taylor's series method for solving IVP $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$. (2M)
 - d) Write the Fourier series for $f(x)$ in the interval $(-\pi, \pi)$. (2M)
 - e) Write the suitable solution of one dimensional heat equation. (2M)
 - f) Write the suitable solution of one dimensional heat equation. (2M)
 - g) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that (2M)
- $$F\left\{f(x) \cos ax\right\} = \frac{1}{2} F(s+a) + F(s-a).$$

PART -B

2. a) Using Regula-Falsi method, compute the real root of the equation $x^3 - 4x - 9 = 0$. (7M)
- b) Develop an Iterative formula to find $\frac{1}{N}$. Using Newton-Raphson method. (7M)
3. a) Evaluate $\frac{x^2}{\cos 2x}$. (6M)
- b) Compute $f(27)$ Using Lagrange's formula from the following table: (8M)

x	14	17	31	35
f(x)	68.7	64.0	44.0	39.1



4. a) Evaluate $\int_0^{0.6} e^{-x^2} dx$ by using Simpson's $\frac{1}{3}$ rd rule taking seven ordinates. (7M)
- b) Given that $\frac{dy}{dx} = 2 + \sqrt{xy}$, $y(1) = 1$. (7M)
- Find $y(2)$ in steps of **0.2** using the Euler's method.

$$x, 0 \leq x \leq \pi \quad (7M)$$

5. a) Find the Fourier series for the function $f(x) = 2\pi - x$, $\pi \leq x \leq 2\pi$.
Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$. (7M)
- b) Obtain the Fourier expansion of $f(x) = x \sin x$ as a cosine series in $(0, \pi)$. (7M)

6. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy -plane, (14M)
 $0 \leq x \leq a$ and $0 \leq y \leq b$ satisfying the following boundary condition
 $u(0, y) = 0$, $u(a, y) = 0$, $u(x, b) = 0$ and $u(x, 0) = f(x)$.

7. a) Find the Fourier sine transform of the function
 x , $0 < x < 1$ (7M)
- $f(x) = 2 - x$, $1 < x < 2$.
- u , $x > 2$
- b) Find the Fourier cosine integral and Fourier sine integral of (7M)
- $f(x) = e^{-kx}$, $k > 0$.



I B. Tech II Semester Regular Examinations, April/May - 2017
MATHEMATICS-II (MM)
 (Com. to CE, EEE, ME, CHEM, AE, BIO, AME, MM, PE, PCE, MET)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **FOUR** Questions from **Part-B**
- ~~~~~

PART -A

1. a) Explain Iteration method. (2M)
 b) Prove that $\alpha = \frac{1}{2} (E^{1/2} + E^{-1/2})$. (2M)
 c) Prove that ${}^3y_2 = \nabla^3 y_5$. (2M)
 d) Explain Runge-Kutta method of fourth order for solving IVP (2M)
 $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.
 e) Write the Fourier series for $f(x)$ in the interval $(-L, L)$. (2M)
 f) Write the various possible solutions of two-dimensional Laplace equation. (2M)
 g) If $F(s)$ and $G(s)$ are the complex Fourier transform of $f(x)$ and $g(x)$, then prove that $F\{af(x) + bg(x)\} = aF(s) + bG(s)$. (2M)

PART -B

2. a) Find a positive real root of the equation $x^4 - x - 10 = 0$ using Newton-Raphson's method. (7M)
 b) Explain the bisection method. (7M)
3. a) Evaluate $\int_0^{\pi/2} \cos 2x \, dx$. (6M)
 b) Using Newton's backward formula compute $f(84)$ from the following table: (8M)

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304



4. a) Evaluate $\int_0^1 e^{-x^2} dx$ by using Trapezoidal rule with $n = 10$. (7M)

b) Obtain Picard's second approximate solution of the initial value problem (7M)

$$\frac{dy}{dx} = \frac{x^2}{y^2 + 1}, y(0) = 0.$$

5. a) Obtain the Fourier series $f(x) = \frac{\pi - x^2}{2}$ in the interval $0 < x < 2\pi$. Deduce that (8M)

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}.$$

b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. (6M)

6. a) Solve by the method of separation of variables (7M)

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y} \text{ and } u(0, y) = 8e^{-3y}.$$

b) Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy -plane, (7M)

$0 \leq x \leq a$ and $0 \leq y \leq b$ satisfying the following boundary condition
 $u(x, 0) = 0, u(x, b) = 0, u(0, y) = 0$ and $u(a, y) = f(y)$.

7. a) Find the Fourier cosine integral and Fourier sine integral of (7M)

$$f(x) = e^{-ax} - e^{-bx}, a > 0, b > 0.$$

b) Find the Fourier transform of $e^{-a^2 x^2}$, $a > 0$. Hence deduce that $e^{-\frac{x^2}{2}}$ is self reciprocal in respect of Fourier transform. (7M)

